with the static characteristic corresponding to the dynamic bias E_c. With triodes, $E_{co} = I_{bo}R_{cc}$ and $E_c = I_{ba}R_{cc}$, where R_{cc} is the biasing resistance. With tetrodes and pentodes the action is more complicated than with triodes, since the bias is produced not only by the average plate current but also by the screen current, which varies with bias and may vary appreciably with excitation.

It should be noted that dependence of a-c load resistance upon frequency causes the slope of the dynamic load line to change with frequency. Furthermore, departure of the plate-current wave from sinusoidal form increases with amplitude of excitation voltage, so that the difference between static and dynamic average plate current also increases with excitation amplitude. The form of the plate diagram therefore changes with frequency and with amplitude of excitation.

In graphical analyses of vacuum-tube circuits the average plate current I_{ba} is determined from the dynamic load line, from the dynamic transfer characteristic, or from the wave of plate current derived from the dynamic load line. Since the value of I_{ba} must be known in order to locate the dynamic load line, the dynamic operating point and dynamic load line must be located by trial. If the dynamic operating point is correctly chosen, the average plate current, as determined from the dynamic load line or the wave of plate current derived from it, must be equal to the current I_{ba} at the assumed dynamic operating point. Location of the dynamic operating point is greatly facilitated by the use of graphical methods of harmonic analysis. For this reason it is desirable to study graphical methods of harmonic analysis before discussing practical methods of locating the dynamic load line and analyzing the plate diagram.

4-11. Graphical Analysis of Plate Current.—Graphical methods of analyzing the plate current of a vacuum tube are based upon the assumption that for sinusoidal excitation voltage the plate current contains a finite number of frequency components, or upon the equivalent assumption that the plate current may be expressed as a finite series. If as many instantaneous values of current can be determined graphically as there are components, or terms in the series, simultaneous equations can be set up from which the amplitudes may be computed. A number of interesting and useful methods have been developed, which differ from one another mainly as to choice of the points of the cycle at which the currents are evaluated and as to whether the instantaneous currents are measured with respect to alternating current zero or with respect to the instantaneous values that would obtain at those instants in the cycle if there were no distortion.[1] The accuracy and convenience of these methods depend upon the manner in which the selected currents are

[1] See bibliography at end of chapter.

chosen. The currents may be evaluated at equal time intervals of the excitation cycle, as exemplified by the method of Lucas; they may be evaluated at equal grid-voltage intervals, as in Espley's method; or they may be evaluated at such instants as to give the highest accuracy. The advantage of the second method lies in the fact that the grid voltages at which the current is evaluated may often be made to coincide with those of the static plate characteristics, so that the currents may be read directly from the intersections of the dynamic load line with these static characteristics. Chaffee has developed a more general method which includes all three of these special methods.[1]

In the treatment that follows, it will be assumed that the load is nonreactive. The path of operation is then a straight line, and evaluating the current at a given number of points on the path of operation is equivalent to evaluating at twice that number of instants in the fundamental cycle. It will sometimes be convenient to differentiate between methods of analysis according to the number of instantaneous values of current that are used to determine the amplitudes of the fundamental and harmonic frequency components. Thus a seven-point analysis is one in which the current is evaluated at seven points of the path of operation or of the dynamic transfer characteristic, or at 14 instants in the fundamental period.

4-12. Derivation of Formulas for Graphical Analysis.—In the following analysis the excitation voltage is assumed to be impressed upon the control grid in order to vary the plate current. The form of the analysis is similar when the excitation is applied to any electrode in order to vary the current of that or any other electrode. With suitable changes of symbols, therefore, the equations derived in this section may be applied to the analysis of the alternating currents flowing in any electrode when the excitation is sinusoidal and the load nonreactive. Since the instantaneous alternating voltage across a nonreactive load is proportional to the instantaneous alternating plate current, the equations may also be applied to the analysis of load voltage. The use of the equations may, in fact, be extended to the harmonic analysis of any quantity that varies periodically as the result of the sinusoidal variation of another quantity that is related to the first through a single-valued curve of known form.

Substitution of the sinusoidal voltage $E_{gm} \sin \omega t$ for the excitation voltage e in the series expansion for i_p [Eq. (3-40)], gives

$$i_p = a_1 E_{gm} \sin \omega t + a_2 E_{gm}^2 \sin^2 \omega t + a_3 E_{gm}^3 \sin^3 \omega t + a_4 E_{gm}^4 \sin^4 \omega t + \cdots \quad (4\text{-}21)$$

which may be written in the form

[1] CHAFFEE, E. L., *Rev. Sci. Instruments*, **7**, 384 (1936).

$$i_p = H_0 + H_1 \sin \omega t - H_2 \cos 2\omega t - H_3 \sin 3\omega t + H_4 \cos 4\omega t + H_5 \sin 5\omega t - \cdots \quad (4\text{-}22)$$

in which H_n is the amplitude of the nth harmonic component of the alternating plate current and H_0 is the steady component of alternating plate current. As previously explained, the time axis of the alternating-plate-current wave passes through the point T, and i_p is measured with respect to the current I_{bt}, corresponding to the time axis. The total instantaneous plate current is

$$i_b = I_{bt} + i_p = I_{bt} + H_0 + H_1 \sin \omega t - H_2 \cos 2\omega t - H_3 \sin 3\omega t + H_4 \cos 4\omega t + H_5 \sin 5\omega t - \cdots \quad (4\text{-}23)$$

Formulas must be derived for the coefficients of Eq. (4-23) in terms of selected values of instantaneous plate current. The accuracy of these

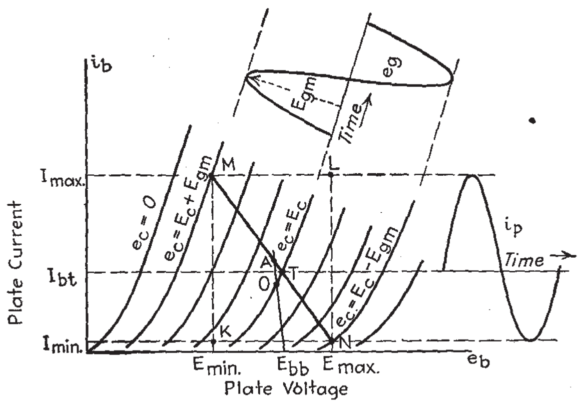

FIG. 4-20.—Diagram used in the derivation and application of Eqs. (4-29).

formulas for harmonic amplitudes increases with the number of points of the fundamental cycle at which the current is evaluated and also depends upon the location of these points in the cycle. The variation of the instantaneous total plate current with amplitude of a given harmonic is greatest at the instants at which the harmonic has its crest value. Highest accuracy is therefore obtained if the currents used to determine the amplitude of a given harmonic correspond to the instants at which the harmonic has its crest value.

In order to explain the method and thus justify the use of the formulas, Espley's formulas will be derived for the simple case in which the third and higher harmonics are assumed to be negligible. Under this assumption the alternating plate current may be expressed by two terms of the plate-current series. Plate excitation voltage is assumed to be zero. The total instantaneous plate current is

$$i_b = I_{bt} + a_1 e_g + a_2 e_g^2 \quad (4\text{-}24)$$

The following relations are apparent from Fig. 4-20:

$$i_b = I_{\max} \quad \text{when} \quad e_g = E_{gm} \tag{4-25}$$

$$i_b = I_{\min} \quad \text{when} \quad e_g = -E_{gm} \tag{4-26}$$

Substituting Eqs. (4-25) and (4-26) in Eq. (4-24) and solving the resulting simultaneous equations gives

$$a_1 = \frac{I_{\max} - I_{\min}}{2E_{gm}} \qquad a_2 = \frac{I_{\max} + I_{\min} - 2I_{bt}}{2E_{gm}^2} \tag{4-27}$$

Substituting Eq. (4-27) in the first two terms of Eq. (4-21) and expanding $\sin^2 \omega t$ gives

$$i_b = I_{bt} + \tfrac{1}{4}(I_{\max} + I_{\min} - 2I_{bt}) + \tfrac{1}{2}(I_{\max} - I_{\min}) \sin \omega t - \tfrac{1}{4}(I_{\max} + I_{\min} - 2I_{bt}) \cos 2\omega t \tag{4-28}$$

The average plate current and the amplitudes of the fundamental and second-harmonic components of plate current are

$$\left.\begin{aligned} H_0 &= \tfrac{1}{4}(I_{\max} + I_{\min} - 2I_{bt}) \\ I_{ba} &= I_{bt} + H_0 = \tfrac{1}{4}(I_{\max} + I_{\min} + 2I_{bt}) \\ H_1 &= \tfrac{1}{2}(I_{\max} - I_{\min}) \\ H_2 &= \tfrac{1}{4}(I_{\max} + I_{\min} - 2I_{bt}) \end{aligned}\right\} \tag{4-29}$$

It is important to note that Eqs. (4-29) will give sufficiently accurate values of steady, fundamental, and second-harmonic components of plate current only when higher harmonics are negligible. If the higher harmonics cannot be neglected, it is necessary to use formulas based upon a greater number of terms of the series expansion for i_p. To derive equations that include harmonics up to the nth, n terms of the expansion are used, and the series is evaluated at $n + 1$ values of instantaneous grid voltage. Espley's method for four harmonics requires the determination of the instantaneous plate currents corresponding to $e_g = +E_{gm}$, $e_g = +\frac{1}{2}E_{gm}$, $e_g = 0$, $e_g = -\frac{1}{2}E_{gm}$, and $e_g = -E_{gm}$. These five values of current will be represented by the symbols I_1, $I_{1/2}$, I_{bt}, $I_{-1/2}$, and I_{-1}, respectively. The following expressions give the amplitudes of the components of plate current:

$$\left.\begin{aligned} I_{ba} &= \tfrac{1}{6}(I_1 + 2I_{1/2} + 2I_{-1/2} + I_{-1}) \\ H_1 &= \tfrac{1}{3}(I_1 + I_{1/2} - I_{-1/2} - I_{-1}) \\ H_2 &= \tfrac{1}{4}(I_1 - 2I_{bt} + I_{-1}) \\ H_3 &= \tfrac{1}{6}(I_1 - 2I_{1/2} + 2I_{-1/2} - I_{-1}) \\ H_4 &= \tfrac{1}{12}(I_1 - 4I_{1/2} + 6I_{bt} - 4I_{-1/2} + I_{-1}) \end{aligned}\right\} \tag{4-30}$$

I_1, $I_{1/2}$, $I_{-1/2}$, and I_{-1} are determined from the intersections of the dynamic load line with the static characteristics corresponding to $e_c = E_c + E_{gm}$, $e_c = E_c + \frac{1}{2}E_{gm}$, $e_c = E_c - \frac{1}{2}E_{gm}$, and $e_c = E_c - E_{gm}$, respectively, as shown

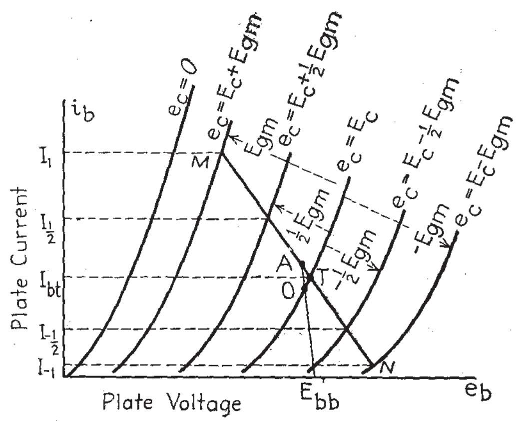

FIG. 4-21.—Use of the plate diagram in the application of Eqs. (4-30).

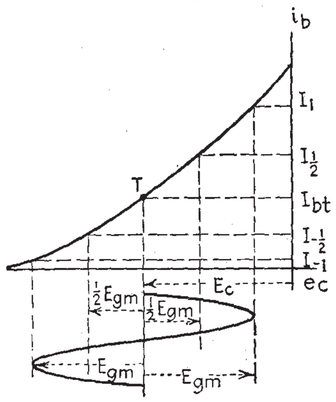

FIG. 4-22.—Use of the dynamic transfer characteristic in the application of Eqs. (4-30).

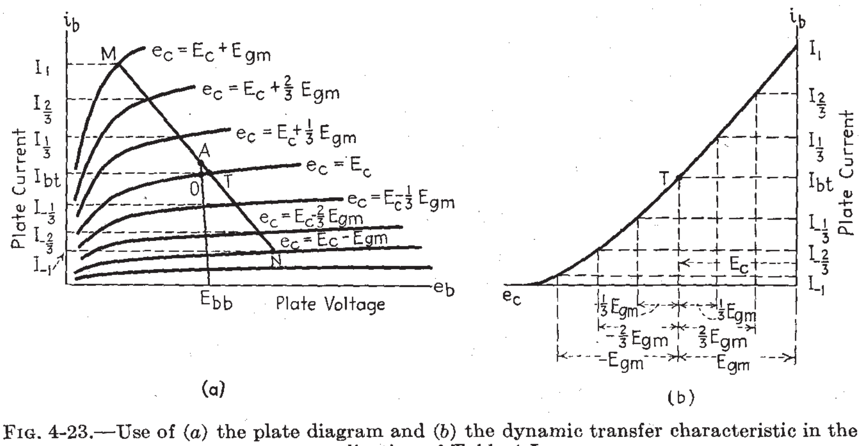

FIG. 4-23.—Use of (a) the plate diagram and (b) the dynamic transfer characteristic in the application of Table 4-I.

in Fig. 4-21. If the static characteristics corresponding to these values of grid voltage are not available, the currents may be read from the dynamic transfer characteristic, as shown in Fig. 4-22.

For six harmonics, Espley's method requires that the plate current be evaluated at the seven points of the load line or dynamic transfer

TABLE 4-I

	I_1	$I_{2/3}$	$I_{1/3}$	I_{bt}	$I_{-1/3}$	$I_{-2/3}$	I_{-1}
I_{ba}	0.131	0.295	−0.105	0.359	−0.105	0.295	0.131
H_1	0.261	0.394	−0.07	0	0.07	−0.394	−0.261
H_2	0.218	0.19	−0.475	0.133	−0.475	0.19	0.218
H_3	0.176	−0.141	−0.246	0	0.246	0.141	−0.176
H_4	0.12	−0.295	0.105	0.141	0.105	−0.295	0.12
H_5	0.063	−0.253	0.316	0	−0.316	0.253	−0.063
H_6	0.032	−0.19	0.475	−0.633	0.475	−0.19	0.032

characteristic corresponding to $e_g = 0$, $\pm \frac{1}{3}E_{gm}$, $\pm \frac{2}{3}E_{gm}$, and $\pm E_{gm}$, as indicated in Figs. 4-23a and 4-23b. The coefficients by which these currents must be multiplied are given in Table 4-I. To find the amplitude of the harmonic listed in any row of this table, the coefficients appearing in that row are multiplied by the graphically determined numerical values of the currents listed at the heads of the columns, and added.

Espley's equations are convenient to use if the instantaneous grid voltages at which the currents are found correspond to the given static characteristics; but the accuracy, particularly of the even harmonics, is not the highest that may be obtained, since the instantaneous currents do not in general correspond to the instants at which the harmonics have crest values. Maximum seven-point accuracy in the evaluation of each harmonic would require a different set of seven instantaneous currents for each harmonic and would therefore greatly increase the work of analysis. The following equations, in which the currents are evaluated at or near the instants at which the harmonics have crest values, have five- and seven-point

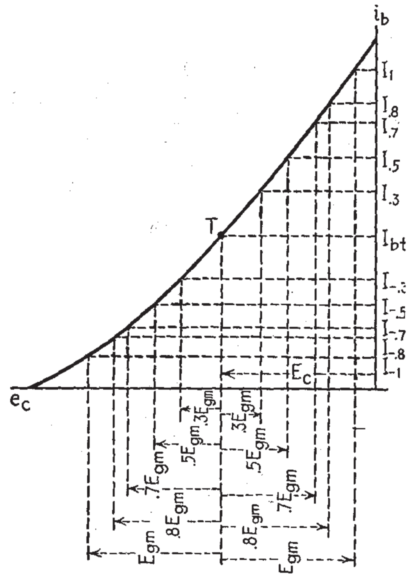

FIG. 4-24.—Use of the dynamic transfer characteristic in the application of Eqs. (4-31).

accuracy. The currents must be found corresponding to plus and minus 0.3, 0.5, 0.7, 0.8, and full grid swing, as shown in Fig. 4-24. If the currents are indicated by the symbols I_1, I_{-1}, $I_{.8}$, $I_{-.8}$, etc., the harmonic amplitudes are

$$\left.\begin{aligned} I_{ba} &= \tfrac{1}{6}(I_1 + 2I_{.5} + 2I_{-.5} + I_{-1}) \\ H_2 &= \tfrac{1}{4}(I_1 - 2I_{bt} + I_{-1}) \\ H_3 &= \tfrac{1}{6}(I_1 - 2I_{.5} + 2I_{-.5} - I_{-1}) \\ H_4 &= \tfrac{1}{8}(I_1 - 2I_{.7} - 2I_{bt} - 2I_{-.7} - I_{-1}) \\ H_5 &= 0.095I_1 - 0.197I_{.8} + 0.207I_{.3} - 0.207I_{-.3} \\ &\qquad + 0.197I_{-.8} - 0.095I_{-1} \\ &\cong 0.1(I_1 - 2I_{.8} + 2I_{.3} - 2I_{-.3} + 2I_{-.8} - I_{-1}) \\ H_1 &= \tfrac{1}{2}(I_1 - I_{-1} - H_3 - H_5) \\ &\cong \tfrac{1}{30}(7I_1 + 6I_{.8} + 10I_{.5} - 6I_{.3} + 6I_{-.3} \\ &\qquad - 10I_{-.5} - 6I_{-.8} - 7I_{-1}) \end{aligned}\right\} \quad (4\text{-}31)$$

These formulas for I_{ba}, H_2, and H_3 are the same as those given under Eqs. (4-30). That for H_4 may be derived by a modification of Espley's method or by that of Chaffee, under the assumption that $\sin 45° = 0.700$. Those for H_5 and H_1 may be derived by Espley's or Chaffee's method for seven points.

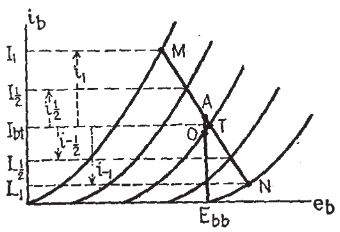

FIG. 4-25.—Diagram showing the relation between instantaneous currents measured relative to zero and relative to I_{bt}.

Sometimes it is convenient or necessary to measure the instantaneous alternating currents (with respect to the current corresponding to the time axis I_{bt}), rather than the instantaneous total currents, in making graphical analyses. The following relations are apparent from Fig. 4-25:

$$\left.\begin{aligned} I_1 &= I_{bt} + i_1 & I_{-\frac{1}{2}} &= I_{bt} - i_{-\frac{1}{2}} \\ I_{\frac{1}{2}} &= I_{bt} + i_{\frac{1}{2}} & I_{-1} &= I_{bt} - i_{-1} \end{aligned}\right\} \quad (4\text{-}32)$$

in which the lower-case symbols indicate currents measured relative to I_{bt}. By means of Eqs. (4-32), Eqs. (4-30) may be transformed into

$$\left.\begin{aligned} I_{ba} &= \tfrac{1}{6}(i_1 + 2i_{\frac{1}{2}} + 6I_{bt} - 2i_{-\frac{1}{2}} - i_{-1}) \\ H_1 &= \tfrac{1}{3}(i_1 + i_{\frac{1}{2}} + i_{-\frac{1}{2}} + i_{-1}) \\ H_2 &= \tfrac{1}{4}(i_1 - i_{-1}) \\ H_3 &= \tfrac{1}{6}(i_1 - 2i_{\frac{1}{2}} - 2i_{-\frac{1}{2}} + i_{-1}) \\ H_4 &= \tfrac{1}{12}(i_1 - 4i_{\frac{1}{2}} + 4i_{-\frac{1}{2}} - i_{-1}) \end{aligned}\right\} \quad (4\text{-}33)$$

If the dynamic transfer characteristic is symmetrical about E_c, then

$i_1 = i_{-1}$ and $i_{1/2} = i_{-1/2}$, and Eqs. (4-33) reduce to

$$\left.\begin{array}{ll} I_{ba} = I_{bt} = I_{bo} & H_1 = \frac{2}{3}(i_1 + i_{1/2}) \\ H_2 = H_4 = 0 & H_3 = \frac{1}{3}(i_1 - 2i_{1/2}) \end{array}\right\} \qquad (4\text{-}34)$$

In a similar manner, for a symmetrical dynamic transfer characteristic, the seven-point equations corresponding to Table 4-I become

$$\left.\begin{array}{l} H_0 = H_2 = H_4 = H_6 = 0 \\ H_1 = 0.522i_1 + 0.787i_{2/3} - 0.141i_{1/3} \\ H_3 = 0.351i_1 - 0.281i_{2/3} - 0.492i_{1/3} \\ H_5 = 0.127i_1 - 0.506i_{2/3} + 0.633i_{1/3} \end{array}\right\} \qquad (4\text{-}35)$$

For a symmetrical characteristic, Eqs. (4-31) are

$$\left.\begin{array}{l} H_1 = \frac{1}{15}(7i_1 + 6i_{.8} + 10i_{.5} - 6i_{.3}) \\ H_3 = \frac{1}{3}(i_1 - 2i_{.5}) \\ H_5 = \frac{1}{5}(i_1 - 2i_{.8} + 2i_{.3}) \end{array}\right\} \qquad (4\text{-}36)$$

Equations (4-34), (4-35), and (4-36) show that with symmetrical dynamic transfer characteristics the dynamic and static operating points coincide and the even harmonics are zero. These equations will be found useful in the analysis of *push-pull* amplifiers, which will be defined in Chap. 5. The dynamic transfer characteristics of push-pull amplifiers are of the form shown in Fig. 4-26. I_{bt} is seen to be zero in this case.

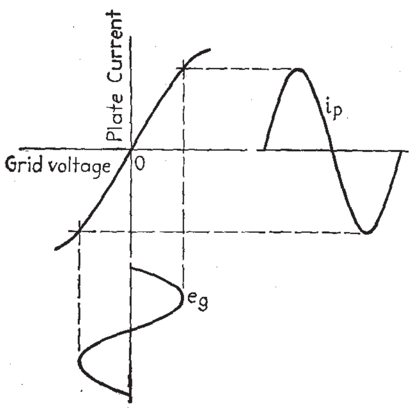

FIG. 4-26.—Symmetrical wave of plate current produced when the tranfer characteristic is symmetrical.

4-13. Choice of Equations.—The choice of the equations to be used in a particular analysis depends largely upon the accuracy required and upon the convenience of application. The number of static plate characteristics intersected by the load line may be such as to make it convenient to apply a seven-point analysis when five-point accuracy is sufficient. For very high accuracy, or in the determination of a large number of harmonics, some form of "schedule" method of analysis may have to be used.[1]

4-14. Field of Application of Equations for Harmonic Analysis.—As pointed out at the beginning of Sec. 4-11, the development of the equations of Secs. 4-11 and 4-12 was made for the special case of excita-

[1] MALTI, M. G., "Electric Circuit Analysis," p. 188, John Wiley & Sons, Inc., New York, 1930; KNIGHT, A. R., and FETT, G. H., "Introduction to Circuit Analysis," Chap. 14, Harper & Brothers, New York, 1943.

tion applied to the control-grid circuit in order to vary the plate current. By suitable changes of symbols the equations may be applied to the analysis of the current or voltage of any other electrode or of any quantity that varies periodically as the result of the sinusoidal variation of another quantity related to the first through a single-valued curve of known form.

4-15. Significance of Algebraic Signs of Numerical Values of Current Components.—It should be noted that the algebraic sign of the amplitude of a particular harmonic may turn out to be either positive or negative. The significance of the algebraic signs is indicated by Eq. (4-22). A reversal of sign merely indicates a 180-degree shift in the phase of a given harmonic. If the amplitude is positive, the harmonic adds to the fundamental at the instant when the fundamental has its positive crest value; if the amplitude is negative, the harmonic subtracts from the fundamental at that instant.

4-16. Nonsinusoidal Excitation Voltage.—The formulas for harmonic content developed in this chapter are all based upon the assumption that the alternating grid voltage is sinusoidal. They are not, therefore, of value in analyzing the alternating plate current when the grid voltage is not sinusoidal. It is possible, however, to construct a wave of plate current by means of the dynamic transfer characteristic and to analyze it by well-known methods of wave analysis.[1] This neglects the fact that the a-c load resistance and slope of the load line may be different for each input frequency component. An approximate analysis may be made under the assumption that the components of the wave of plate current are the same as would be obtained if the various components of the grid voltage were applied separately and the corresponding output components added. This would be valid if there were no intermodulation. It gives no indication of the intermodulation frequencies and neglects the fact that the dynamic operating point is different for each component than it would be for the resultant grid voltage. Ordinarily, adequate indication of the performance of the tube and circuit may be obtained by graphical methods based upon sinusoidal excitation, with full excitation voltage.

4-17. Mechanical Aids to Harmonic Measurement.—Scales that make possible the direct reading of percentage second and third harmonic have been described by D. C. Espley and L. I. Farren.[2] These scales, which are based upon a five-point analysis, are particularly useful when a large number of graphical determinations of harmonic content must be made.

[1] Moullin, E. B., *Wireless Eng.*, **8**, 118 (1931).

[2] Espley, D. C., and Farren, L. I., *Wireless Eng.*, **11**, 183 (1934); Sarbacher, R. L., *Electronics*, December, 1942, p. 52.